Midterm 2 Review Problems

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1 structure theorem and modules

Problem 1.1 (Source: Iz)**.** Determine $\mathrm{Hom}_{\mathbb{Z}}(G, \mathbb{Z}/5\mathbb{Z})$ and $\mathrm{Hom}_{\mathbb{Z}}(G, \mathbb{Z}/4\mathbb{Z})$ for every abelian group of order 20.

2 symmetric group, class equation, group actions

Problem 2.1 (Artin Exercise 7.5.2)**.** What is the centralizer of the element (12) in S_5?

Problem 2.2 (Dummit and Foote Exercise 4.4.13)**.** Let G be a group of order 203. Prove that if H is a normal subgroup of order 7 in G, then $H \subseteq Z(G)$. Deduce that G is abelian in this case.

Problem 2.3 (Artin Exercise 7.2.2)**.** A group of order 21 contains a conjugacy class $C(x)$ of order 3. What is the order of x in the group?

Problem 2.4 (Artin Exercise 7.2.3)**.** A group of order 12 contains a conjugacy class of order 4. Prove that the center of G is trivial.

3 linear algebra

We went over the following statements:

Proposition 3.1 (Artin Corollary 4.7.13)**.** Let T be a linear operator on a finite-dimensional complex vector space. The following conditions are equivalent:

(a) T is a diagonalizable operator,

(b) every generalized eigenvector is an eigenvector,

(c) all of the blocks in the Jordan form for T are 1×1 blocks.

Proposition 3.2 (Artin Theorem 4.7.14)**.** Let T be a linear operator on a finite-dimensional complex vector space V. If some positive power of T is the identity, say $T^r = I$, then T is diagonalizable.

For Prop 3.2, I may have forgotten to specify that V was a vector space over $\mathbb{C}$ (very necessary). Sorry.

Problem 3.3 (Axler 8.D.5)**.** Suppose $T : V \to V$ is a linear operator and $v_1, \ldots, v_n$ is a basis of V that is a Jordan basis for T. Describe the matrix of T^2 with respect to this basis.

Problem 3.4 (Axler 7.A.17)**.** Suppose $T : V \to V$ is self-adjoint. Prove that $\ker T^k = \ker T$ and $\operatorname{im} T^k = \operatorname{im} T$ for every positive integer k.